

PRE-(CALCULUS

For Grade 11 Senior High School
Science, Technology, Engineering, and
Mathematics (STEM)

Joel T. Lao

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